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NONLINEAR MODES FOR A DISCRETE MECHANICAL SYSTEM WITH RIGID CONTACT

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Nonlinear normal modes (NNM) [2, 3] are useful tools to study nonlinear vibrations for mechanical systems. We are interested in finding NNM in the context of perfectly elastic unilateral contact. To this end is investigated a discrete model representing a flexible thin rod in contact against a frictionless rigid foundation [4]. This one dimensional model is essentially given by coupled harmonic oscillators with an unilateral rigid contact against a wall. We assume that the contact preserves energy. A natural mathematical formulation is derived and important features for this N dof system are explored: grazing contact, periodic solutions.

Let us give a flavor of our results for the simple 2 dof case: (u_1, v_1, u_2, v_2) belonging to the phase space $\mathbb{R}^4$ and $v_i = \dot{u}_i$. Families of periodic trajectories are computed and the associated non-smooth invariant manifolds are investigated. The problem is clearly

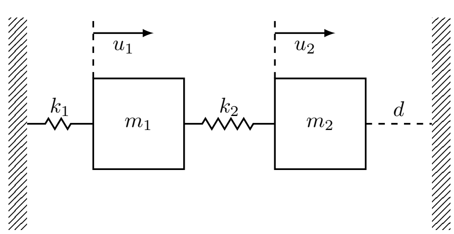

Figure 1: Discrete model with $N = 2$ dof

nonlinear but we use a linear procedure introduced in [5] and generalized for N dof, $N > 2$. The vibratory energy is continuously increased in order to reach grazing as well as transverse unilateral contact conditions. The corresponding autonomous periodic orbits

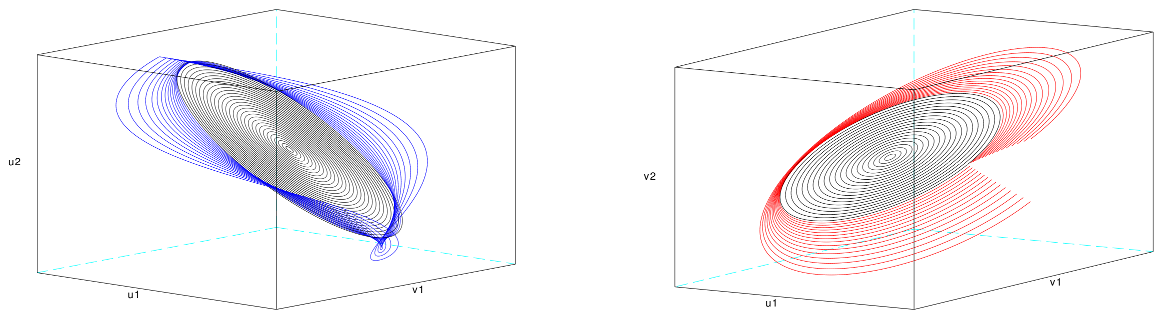

Figure 2: Invariant manifolds supporting autonomous periodic trajectories in variables (left) (u_1, v_1, u_2) and (right) (u_1, v_1, v_2)

are organized on non-smooth invariant manifolds of dimension 2 in the state space of the system of interest.

This applied mathematical subject has many applications and opportunities in Mechanical Engineering, in particular, in aeronautic where unilateral contact occurrences are becoming common due to the use of more flexible and lighter structures implying larger displacements.

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